Cantaluppi & Hug AG
Software and
Consulting

Practical Performance Calculation

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Target audience

This document is intended not only for people who need to calculate the performance of financial investments, but also for people who need to analyze and interpret performance reports of financial investments: CFOs, foundation boards, analyst advisors and investors themselves.

Although this document contains some mathematical formulas, they are not essential for understanding the material. The description of the cases and the numerous examples provide a sufficient instruction.

It is not the aim of this document to present all methods of performance calculation, but to show the most common methods, with their practical difficulties and the risks of a deceptive interpretation of their results.

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Introduction

This document emphasizes the practical aspect of performance calculation. We do not develop long theories and do not present all known computational methods. We restrict ourselves to a subjective choice of practical methods and show how to interpret their results. We show many examples, with non-intuitive situations and caveats to avoid. This document therefore addresses people confronted with the interpretation of the results of performance reports.

We start by defining capital flows since they tremendously influence the computation of the performance. After analyzing the calculation of performance in absence of capital flows, we examine the TWR and MWR performance. We show how to interpret and compare them and show some examples that at first surprise or even look erroneous.

Capital Flows

A performance is always computed for a given period. The data needed to calculate the MWR performance are the value of the investment at the beginning and at the end of the period and the capital flows during the period. In order to compute the TWR performance the value of the investment at the times of the capital flows are additionally needed.

In a first step we analyze the calculation of performance without capital flows. After the initial capital is invested, there is no subsequent deposit or withdrawal of capital during the period of the performance measurement, neither as securities nor as cash. In this case the performance depends solely on the value of the investment at the beginning and at the end of the computation period. What happens in between is irrelevant for the performance result, it can only be internal transactions.

In a second step we calculate the performance in the presence of capital flows (deposits or withdrawals) during the computation period. Of course in this case the values of the investment at the beginning and at the end of the computation period do not suffice for the calculation of the performance. We will then see that there are several methods to compute the performance that may lead to different results. Which one to use depends on the "type of performance" that has to be presented.

Note that a capital flow is dependent on the investment that is analyzed. A purchase of stocks, for example, does not generate a cash flow at the level of the whole investment since it simply is a transfer of capital from the liquidities towards the stocks, but it is an inflow of capital when considering the stocks sub portfolio of the investment.

Without Capital Flows

Simple Performance

Suppose that an investment has a value of CHF 210 as of 31.12.2012 and a value of CHF 217.35 as of 31.12.2013. There was no external capital flow during the whole year. What is the performance of this investment for the year 2013? This performance is given by

(217.35 - 210) / 210 = 0.035 = 3.50 %

Let us now see the general formula for such a performance calculation. Let V_b the value of the investment at the beginning of the period and V_e the value of the investment at the end of the period. The performance P for the period considered is given by

$$P = \frac{V_e - V_b}{V_b}$$

Notice that the value of the investment considered here is its market value, including the accrued interests.

Performance of a Combination of Investments

An investment consists in two portfolios A and B. Portfolio A has a value of CHF 200 as of 31.12.2012 and a value of CHF 205 as of 31.12.2013, which gives a performance of (205 - 200) / 200 = 2.5%. Portfolio B has a value of CHF 300 as of 31.12.2012 and a value of CHF 312 as of 31.12.2013, which gives a performance of (312 - 300) / 300 = 4%. The whole investment has a value of CHF 500 (200 + 300) as of 31.12.2012 and a value of CHF 517 (205 + 312) as of 31.12.2013, which gives a performance of (517 - 500) / 500 = 3.4%.

The performance of the whole investment can also be computed as a weighted sum of the performances of the portfolios A and B, the weights being given by the relative value of the investment at the beginning of the period, i.e. as of 31.12.2012. We thus have ((200 x 2.5) + (300 x 4)) / (200 + 300) = 3.4%.

The situation is summarized in the following table:

	Initial value 31.12.2012	End value 31.12.2013	Performance in %
Sub portfolio A	200	205	2.50
Sub portfolio B	300	312	4.00
Whole investment	500	517	3.40

Let V_b^A the value of portfolio A at the beginning of the period and V_e^A its value at the end of the period. Similarly let V_b^B the value of portfolio B at the beginning of the period and V_e^B its value at the end of the period. Let P^A the performance of portfolio A for the period, i.e.

$$P^A = \frac{V_e^A - V_b^A}{V_b^A}$$

Let P^B the performance of portfolio B and P the performance of the whole investment. We then have

$$P=\frac{\left(V_e^A+V_e^B\right)-\left(V_b^A+V_b^B\right)}{\left(V_b^A+V_b^B\right)}=P^A\frac{V_b^A}{\left(V_b^A+V_b^B\right)}+P^B\frac{V_b^B}{\left(V_b^A+V_b^B\right)}$$

The performance P can be either computed using the formula for the simple performance or as the weighted sum of the performances P^A and P^B of portfolios A and B, the weights being given by the relative weight (relative value) of the corresponding portfolio. The formula can be easily generalized to a larger number of portfolios.

Multi-Period Performance within a year

An investment has a performance of 6% for the period 31.12.2012 to 30.06.2013 and a performance of 4% for the period 30.06.2013 to 31.12.2013. What is the performance of the investment for the year 2013? This performance is given by (1 + 0.06) x (1 + 0.04) - 1 = 0.1024 = 10.24%. Notice that the performance for 2013 is not simply the sum of the performances for the 1^{st} and 2^{nd} half of the year; let us see why. Suppose that the initial investment, as of 31.12.2012 is CHF 100, it then has a value of CHF 106 as of 30.06.2013. This sum is in turn invested and returns a value of CHF 110.24 (= 106 x 1.04) as of 31.12.2013. This non additional combination of performances is called geometric linking or compounding.

In particular, if an investment realizes a performance of +10% in the first half of one year and a performance of -10% in the second half, its yearly performance is -1%, and not 0%. A negative performance must be compensated by a positive performance of larger absolute value in order to recover the initial investment.

We can generalize the method to an arbitrary number n of sub periods. Suppose that the performance of the investment for sub period i is p_i. Then the performance P for the whole period is given by

$$P=\prod_{i=1}^{n}\left(1+p_i\right)-1$$

Multi-Period Performance of a Combination of Investments

Consider an investment consisting of portfolios A and B for the year 2013. The performance results are given for each half-year in the following table.

Invest.	Value 31.12.12 in CHF	Perf. 1^{st} half year in %	Value 30.06.13 in CHF	Perf. 2^{nd} half year in %	Value 31.12.13 in CHF	Perf. year in %
A	200.00	5.00	210.00	3.00	216.30	8.15
B	300.00	-5.00	285.00	7.00	304.95	1.65
Total	500.00	-1.00	495.00	5.30	521.25	4.25

The performance of the whole investment for the year 2013 can be calculated in three different ways:

1. From the values of the whole investment at the beginning and at the end of the year, using the formula for the simple performance calculation: (521.25 - 500) / 500 = 4.25%.
2. As a weighted sum of the performances of portfolios A and B for the year 2013: ((200 x 8.15%) + (300 x 1.65%)) / 500 = 4.25%.
3. By linking geometrically the performances of the whole investment for the two halves of the year 2013: ((1 - 0.01) x (1 + 0.053)) - 1 = 4.25%.

The results are identical, corresponding to our expectations!

Annualization

For relatively long periods (longer than 1 year) the performance is often given in an annualized form, which provides an easier interpretation and comparison of the results. Such an annualization must take into account the geometric character of performance. If a performance P is measured for a period of T days, the annualized performance p_A is given by solving the equation

$$P=\left(1+p_A\right)^{T/365}-1$$

which gives the formula

$$p_A=\left(1+P\right)^{365/T}-1$$

For example, if an investment resulted in a performance of 12.23% over 3 years, its annualized performance is 3.92%.

We use the standard « 365 days / year » that is normally used for annualization of performance figures.

Note that a performance realized over a period shorter than 1 year should not be annualized, since it could be interpreted as a projection of past performance in the future. Such an annualization is prohibited in the Global Investment Performance Standards (GIPS).

With Capital Flows

MWR vs. TWR

The performance of an investment without capital flows is clearly defined and practically all calculation methods lead to the same result, also called return. This is not the case in the presence of capital flows. The TWR (Time Weighted Rate of Return) methods « neutralize » the capital flows whereas the MWR (Money Weighted Rate of Return) methods take them into account in the computation, i.e. a period with a larger investment has a greater weight than a period with a smaller investment. But before going further into the analysis let us look at a concrete example clearly showing the difference between these two methods.

Example

An asset manager invests the money of all his clients in the same way during two subsequent years and obtains a performance of 4% for the first year and 16% for the second year. To make the example as simple as possible assume that the performance only results from market trends, without external capital flows. We now examine the performance of two investors A and B having very different capital flows.

- Investor A invests CHF 1'000'000 at the beginning of the first year and withdraws CHF 900'000 at the beginning of the second year.
- Investor B invests CHF 100'000 at the beginning of the first year and brings another deposit of CHF 900'000 at the beginning of the second year.

The performance of the asset manager given above leads to values at the end of the second year of CHF 162'400 for investor A and CHF 1'164'640 for investor B. Clearly the annual performances are identical for both investors, 4% for the first year and 16% for the second year. However, the MWR performances differ radically for investors A and B, because investor A has realized a performance of 4% on a relatively high investment and a performance of 16% on a relatively low investment, which is the exact opposite for investor B. So for an equal average invested capital of CHF 550’000 investor A has realized a nominal performance (added value) of CHF 62’400 and investor B a nominal performance (added value) of CHF 164’640. Of course the TWR performances are identical for both investors since they only depend on the performance of the asset manager. The relative size of the invested amounts during the two periods does not play any role.

The following table shows the details of the calculation, first for investor A and then for investor B. The first row for each investor shows the calculations with the effective performance (4% the first year, 16% for the second year). The second row shows the result of the computation with annualized MWR, which are 5.4070% for investor A and 14.7690% for investor B. As expected, the values of the portfolios at the end of the second year are identical in both cases.

	1st year				2nd year		
	begin	perform.	end	flow	begin	perform.	end
A real	1'000'000	4.0000	1'040'000	-900'000	140'000	16.0000	162'400
A MWR	1'000'000	5.4070	1'054'070	-900'000	154'070	5.4070	162'400
B real	100'000	4.0000	104'000	900'000	1'004'000	16.0000	1'164'640
B MWR	100'000	14.7690	114'769	900'000	1'014'769	14.7690	1'164'640

This example clearly shows the two points of view: the performance as viewed by the asset manager and the performance as viewed by the investor. Note that this example is rather extreme. The TWR and MWR performances do not differ so drastically under “usual” circumstances.

Since the asset manager is usually not responsible for the external capital flows (deposits or withdrawals by the client), the TWR performance that neutralizes these capital flows is the method of choice to evaluate the performance of the asset manager, at least at the level of the whole investment. Only the TWR performance may be compared to that of other investments (peer group) or to a benchmark. On the other hand, it is a legitimate request from the investor to know the performance of his investment which is given by the MWR performance. This performance is therefore unavoidable but cannot be compared to another investment or a benchmark. The logical consequence is that both TWR and the MWR performances of the whole investment must be presented.

The situation is not as clear cut concerning the performance of sub portfolios of the whole investment, for example the « stocks » part of the investment. In fact, a large part of the capital flows of a sub portfolio (for example the purchase and sale of stocks for the sub portfolio « stocks ») are internal flows that usually result from decisions taken by the asset manager, for example a reallocation from bonds to stocks. These internal flows can be high even when the external flows are negligible. The sole presentation of the TWR performance of sub portfolios can therefore provide a distorted view of the reality, as the following example shows.

Example

Consider the example of an investment that is (for reasons of simplicity) allocated in three asset classes: stocks, bonds and liquidities. The details are given in the table below. A sum of CHF 100’000 is invested at the beginning of the year according to the following allocation: CHF 15’000 in stocks, CHF 15’000 in bonds and CHF 70’000 in liquidities. The performances for the first half of the year are -5% for stocks, -3.5% for bonds and 1.2% for liquidities, allowing us to compute the values of the investment in the three asset classes as of 30. June. A reallocation is then operated, bringing an allocation of CHF 50’000 in stocks, CHF 30’000 in bonds and the remainder of CHF 19’565 in liquidities. Notice that there is neither an increase nor a decrease of capital, it is a pure reallocation, with only internal flows being generated. The performances for the second half of the year are 8% for stocks, 3% for bonds and 1.2% for liquidities, which lead to the values of the investment in the three asset classes given in the table below. The annual performances of the three asset classes correspond to our expectations, but the performance of the whole investment of 4.7% might surprise since it is higher than the performance of each one of the three asset classes!

	Stocks	Bonds	Liquidities	Total
Value beginning of year	15'000.00	15'000.00	70'000.00	100'000.00
TWR 1st half year in %	-5.00	-3.50	1.20	-0.44
Value 30.06	14'250.00	14'475.00	70'840.00	99'565.00
Reallocation	35'750.00	15'525.00	-51'275.00	0.00
Value 01.07	50'000.00	30'000.00	19'565.00	99'565.00
TWR 2nd half year in %	8.00	3.00	1.20	5.16
Value end of year	54'000.00	30'900.00	19'799.78	104'699.78
TWR year in %	2.60	-0.61	2.41	4.70

This surprising result is essentially due to the enormous reallocation that occurred at mid-year. More than CHF 35’000 of liquidities have been invested in stocks and more than CHF 15’000 in bonds, both of these asset classes realizing an appealing performance for the second half of the year, in particular the stocks with a performance of 8%. These two asset classes have had large inflows at mid-year, inflows that are per definition neutralized by the TWR performance. Let us repeat that these flows are entirely due to the reallocation, there has been no increase or decrease of capital at the level of the whole investment. The question arises however of the significance of a performance calculation that neutralizes the flows. If the reallocation is a decision of the asset manager, the performance from the point of view of the asset manager should take them into account. If, however, the reallocation was imposed upon the asset manager by his client, the performance of the asset manager must neutralize these flows. There is therefore no "universal" performance measure.

We also computed the MWR performance of the three asset classes and of the whole investment and obtained the following results:

	Stocks	Bonds	Liquidities	Total
MWR year in %	9.97	1.65	2.43	4.70

This table clearly illustrates the preponderant role of the stocks in the global performance. Notice that the global MWR and TWR performances are identical since there is no capital flow at the level of the whole investment.

The TWR performance has for a long time been considered as THE performance to be presented, because it is the result that can be compared to other performances or benchmarks. It is however becoming obvious that the TWR performance does not always correspond to the perception of the investor and the MWR performance appears to be gaining in popularity.

Let us now see how these performance values are computed.

TWR

The TWR has to neutralize all the capital flows. To this end we divide the performance period into smaller sub periods in which there is no capital flow, compute the performance for each such sub period with the simple performance formula and link these values geometrically on the whole period. In other words, we compute the performance from the beginning of the period to the first capital flow, then from each capital flow to the next capital flow and finally from the last capital flow to the end of the period. The important part is to consider the start value of a sub period including the corresponding capital flow and the end value of a sub period excluding the corresponding capital flow. We have seen earlier how to geometrically link the performances of the sub periods in order to obtain the performance for the whole period.

Let us see how to write this as a mathematical formula. Let $t_0=0$ the beginning of the period of the performance measure, $t_{n+1}=T$ the end of the period and t_i the time point when the capital flow F_i occurred, for i = 1 to n. Let V_i the value of the investment at the time t_i of the capital flow, including this flow. The performance p_i of the investment for the subperiod t_i, t_{i+1} is given by

$$p_i = \frac{V_{i+1} - F_{i+1} - V_i}{V_i}$$

The performance of the investment over the whole period is then given by

$$P = \prod_{i=0}^{n} (1 + p_i) - 1 = \prod_{i=0}^{n} \frac{V_{i+1} - F_{i+1}}{V_i} - 1$$

Example

We calculate the TWR of an investment for the year 2013, i.e. from 31.12.2012 to 31.12.2013. The beginning value of the investment as of 31.12.2012 is CHF 120, there is a withdrawal of CHF 10 on 14.05.2013 and a deposit of CHF 5 on 05.08.2013, leading to an end value of CHF 122 as of 31.12.2013. These data are summarized in the following table:

	Date	Value
Initial value	31.12.2012	120
Outflow	14.05.2013	-10
Inflow	05.08.2013	5
End value	31.12.2013	122

As we have seen above these data are not sufficient to compute the TWR, we also need the value of the investment right before the corresponding capital flows. These values are CHF 126 right before the withdrawal of CHF 10 and CHF 112 right before the deposit of CHF 5. The following table summarizes these data:

	31.12.2012	14.05.2013	05.08.2013	31.12.2013
Value before the flow	120	126	112	122
Flow		-10	5	
Value after the flow		116	117	

The performance from 31.12.2012 to the flow of 14.05.2013 is given by (126 - 120) / 120 = 5.00%. The performance from the flow of 14.05.2013 to the flow of 05.08.2013 is given by (112 - 116) / 112 = -3.45% and the performance from the flow of 05.08.2013 to the end of the period as of 31.12.2013 is given by (122 - 117) / 117 = 4.27%. We can then compound these performances according to the multi-period formula seen earlier in order to obtain the performance for the year 2013, which is given by (1 + 0.05) x (1 - 0.0345) x (1 + 0.0427) - 1 = 5.71%. The TWR performance of this investment for the year 2013 is therefore 5.71%.

We now take a look at how the TWR performance is computed in practice. The value of the investment is not computed at the very moment a cash flow occurs. As a general rule (with exceptions) all the transactions at one particular day are considered having occurred at the end of the day, including the cash flows. The investment is therefore revalued, if necessary, at the end of each day, and we then talk about "True TWR". Such a valuation is not without effort, since it supposes that all security and currency prices of the investment are known at the valuation date. This effort is sometimes too time and resource consuming and the TWR performance is then calculated approximately. Most often the monthly performance is directly calculated like an MWR performance, with the method shown later in this document. The advantage of such a procedure is that security and currency prices are only needed at the month's ends. The approximation can be improved by revaluating the investment at dates where the capital flows are relatively large. One rule accepted in practice is to revaluate the investment when the capital flow exceeds 10% of the value of the investment.

If the performance is also computed at the levels of the asset classes, one should bear in mind that the internal capital flows (between the asset classes) can be much higher than the external capital flows.

Notice that the periodicity of the calculation is not necessarily identical with the periodicity of the presentation, which is often monthly.

MWR

The MWR performance is usually calculated as the internal rate of return (IRR) of the investment. The internal rate of return is relatively difficult to calculate but easy to interpret intuitively. Suppose that the initial investment as well as all the flows (deposits and withdrawals) occur on an account that has a constant interest rate. What should be the interest rate if the account balance at the end of the period were to be equal to the value of the investment at that date? This constant interest rate is the internal rate of return.

Let us first look at the formal definition of the internal rate of return r. In order to make the formula as simple as possible we consider the initial value of the investment V_0 as an initial flow, i.e. $F_0=V_0$. The internal rate of return is the solution to the equation

$$\sum_{i=0}^{n} F_i (1+r)^{T-t_i} = V_e$$

The internal rate of return cannot be calculated directly and must be obtained by successive approximation. The calculation is for example available in MS Excel with the function XIRR, which calculates the internal rate of return as a function of the initial value, the final value and the capital flows and their respective dates. Notice that the end value must be a negative value (like a withdrawal) and that the result in Excel is always annualized.

Example

Let us go back to the example we used above to illustrate the calculation of the TWR performance, whose values and flows are repeated in the table below.

	Date	Amount
Initial value	31.12.2012	120

Withdrawal	14.05.2013	-10
Deposit	05.08.2013	5
Final value	31.12.2013	122

As opposed as with the TWR performance, the values of the investment at the time of capital flows does not influence the MWR performance and is therefore irrelevant. We have computed the internal rate of return (IRR) and obtained a value of 6.05%, which we validate in the following table. We show in this table that the sum of the deposits (the initial value is considered as a deposit) and of the (negative) withdrawals, computed with their respective interests on 31.12.2013 with an interest rate of 6.05% equals CHF 122, which is identical with the value of the investment at that date.

Date	Amount	Number of days of interest	Value 31.12.2013
31.12.2012	120	365	127.26
14.05.2013	-10	231	-10.38
05.08.2013	5	148	5.12
Total			122.00

We see that this investment has an MWR performance of 6.05%, differing slightly from the TWR performance of 5.71% calculated above.

Benchmarks

The calculation of the performance of an investment aims to analyze the results of this investment. To assess this performance, it must be compared with an appropriate benchmark. A benchmark can be an index calculated on the basis of rules based on stock market prices or a weighted average of performances effectively realized by a peer group. For Swiss pension funds, for example, the BVG Pictet indices are calculated on the basis of other indices, whereas the Credit Suisse Pension Fund Index is calculated as the weighted average of the effective performances of a group of pension funds.

If your investment does not belong to a recognized peer group such as pension funds and this investment is not limited to a single asset class, you will probably not find a suitable standard benchmark. You will then have to define your own benchmark. To do this, you must define a composite benchmark as the weighting of standard benchmarks. The performance of the composite benchmark on a given period is then calculated as a weighted average (by the given weighting) of the performances of the standard benchmarks for that period. Let's look at a concrete example with three asset classes: "Liquidity", "Bonds" and "Stocks", each with a given standard benchmark. These benchmarks must be converted into the base currency of the portfolio if they are in another currency. Let us assume a weighting of 15% for "liquidity", 35% for "bonds" and 50% for "stocks". The performance of the composite benchmark for a given period as a function of the performances of the 3 standard benchmarks over the same period is illustrated in the following table:

	Start Holdings	End Holdings	Perf.	Weight in %	Perf. Contribution
Benchmark Liquidity	8'646	8'812	1.9200	15	0.2880
Benchmark Bonds	1'278	1'234	-3.4429	35	-1.2050
Benchmark Stocks	2'073	2'120	2.2672	50	1.1336
Composite Benchmark			0.2166		

The performance of the composite benchmark for this period is 0.2166%.

Usually the weighting of the composite benchmark is assumed to be constant over a certain period, in contrast to a weighting at the beginning of the period that varies with the development of the benchmark value. Note that each calculation of the performance of the composite benchmark means a reallocation of asset classes. The periodicity of the performance calculation then influences the performance of the composite benchmark. A daily performance calculation means a daily reallocation of asset classes towards the given weighting, whereas a monthly calculation means a monthly reallocation.

How are asset class benchmarks determined and how are these benchmarks weighted in composite benchmarks? These parameters are normally derived directly from the investment strategy of the portfolio. The weighting corresponds exactly to the weighting of the investment strategy and the benchmarks are determined for each asset class of this strategy. Note that the benchmark of an asset class itself may be a composite benchmark.

As we will see later, it is possible and desirable to calculate the performance of the various asset classes in a portfolio. These performances can then be compared with the corresponding benchmarks.

One final comment regarding the selection of benchmarks: Since we calculate the performance including interest and dividends of the positions in the portfolio, these should also be taken into account in the selected benchmark. It is therefore important to choose a "total return index" as a benchmark.

Performance of “special” holdings

Some particular holdings give a less intuitive – nonetheless correct - result when it comes to their performance calculation. Some other holdings require their own rules in order to be able to calculate their performance. We analyze here certain special cases and suggest rules that can also be applied to other types of holdings.

Negative holdings (short)

We will now examine the performance of negative holdings (short positions). Suppose that we sell 100 stocks short to a price of CHF 20, which gives us an overdrawn market value of CHF 2’000. Let’s assume that the stock’s price falls to CHF 18. From our initial market value of CHF -2'000 we now have a final market value of CHF -1’800. By applying the simple performance calculation formula, we get the result:

$$p=\frac{-1'800-(-2'000)}{-2'000}=\frac{200}{-2'000}=-10\%$$

This result seems surprising at first since this “investment” in absolute value gave us a profit of CHF 200 and the expected performance is positive. However, the result of a simple investment is calculated by multiplying this investment with the performance. If the result is positive, the performance has to be negative since the investment itself is negative. For a negative investment we will therefore have a negative performance for a positive result and a positive performance for a negative result. This strict application of the performance calculation formula allows to validate the formula that calculates the performance of a combination of investments, as the following example demonstrates:

Example

At the beginning of the period for which the performance is calculated we have a stock holding with a value of CHF 2’000 and a negative holding in calls on those stocks with a value of CHF -300 for a total investment value of CHF 1’700. At the end of the period the value of the stocks dropped to CHF 1’900 and the value of the calls is CHF -240 reflecting the leveraging effect presented by the options. The final value of the investment is CHF 1’660. The following table summarizes this information and gives their performances.

	Initial Value	End Value	Performance in %
Stocks	2'000	1'900	-5
Calls	-300	-240	-20
Total	1'700	1'660	-2.35

The performance of the total investment agrees with the formula for the calculation of the performance for a combination of investments as seen previously. Concretely, we have (2'000 x (-5%)) + ((-300) x (-20%)) / (2'000 - 300) = -2.35%.

FX Forwards

A forward currency contract is a transaction where two currencies are exchanged for an agreed amount at an agreed future date (maturity date). The exchanged amounts set the implicit exchange rate of the contract, which is the forward rate. On the maturity date, the forward contract results in a gain or a loss, depending on whether the spot exchange rate on that day is lower or higher than the exchange rate of the contract. We see that the calculation of the performance according to the simple formula is not possible since the initial investment is zero. It only consists of an agreement which results in a gain or a loss on the maturity date.

Here an example: On the 04.12.2012 a contract is made to sell USD 2'000 against CHF 1'838 on the 06.03.2013. The implicit exchange rate is therefore USD / CHF 0.919. However, on the maturity date the 06.03.2013, the spot exchange rate is USD / CHF 0.941. We therefore sell the USD 2'000 with a value of CHF 2'000 x 0.941 = 1'882 for CHF 1'838 and realize a loss of CHF 44, without having invested any capital. This reflects an even stronger leverage effect than the effect seen previously.

To calculate the performance of a forward currency contract we consider an equivalent investment regarding its result. We invest in cash an amount in the bought currency in order to reach the agreed amount set by the contract on the maturity date, including the interest received, and we borrow an amount in the sold currency in order to reach the agreed amount set by the contract on the maturity date, including the due interests. The bought position is positive, the sold position is negative. The equivalence of this investment with the forward currency on the maturity date is clear since we get exactly the amounts fixed in the forward contract. We can show that this equivalence is valid at any time between the "opening" of the forward and its maturity date.

We now calculate the performance of the forward currency contract as a combination of the positive position ("long") in the purchased currency and of the negative position ("short") in the sold currency as defined above. Since the value of a forward is of zero on its opening date, the absolute value of the initial investment is the same for both holdings, once positive and once negative. We can define the performance of the forward as the performance of the positive position minus the performance of the negative position, according to the formula for the calculation of the performance for a combination of investments. Note that it is imperative to calculate the performance of the negative holding as previously explained. The invested capital is considered the absolute value of the initial capital for both positions.

The next example shows that the combination of a holding in foreign currency and of a forward currency contract used as a partial currency overlay can be perfectly analyzed this way.

Example

We have an investment A of USD 1'000'000 as of 31.12.2012 with the partial currency overlay of USD 750'000 as a forward currency contract. The spot exchange rate as of 31.12.2012 is of USD/CHF 0.95. We commit in a forward contract to sell USD 750'000 as of 31.12.2013 against CHF 704'632, which sets an implicit forward rate of approx. 0.9395. The end exchange rate is a result of the spot rate and of annual interests of 1.875% for the USD and of 0.75% for the CHF. We will now calculate the performance of the total investment and its components for 2013.

As of 31.12.2013 the investment A has a value of USD 1'050'000 and the spot exchange rate USD/CHF is of 0.973. All the results on that date are presented in the following table:

Investment	Ccy.	Value as of 31.12.2012	Value as of 31.12.2013	Perform. in %
Investment A	USD	1'000'000	1'050'000	5.0000
	CHF	950'000	1'021'650	7.5421

- Forward USD part (short)	USD	-736'196	-750'000	
	CHF	-699'387	-729'750	4.3414
- Forward CHF part (long)	CHF	699'387	704'632	0.7500
Total forward	CHF	0	-25'118	-3.5914
Total investment	CHF	950'000	996'525	4.8981

The borrowed amount of USD 736'196 will have a value of USD 750'000 as of 31.12.2013, including interests with a rate of 1.875%. The invested amount of CHF 699'387 will have a value of CHF 704'632 as of 31.12.2013, including interests with a rate of 0.75%. Note that the amount of USD 736'196 is equal to the amount of CHF 699'387 with the spot exchange rate of USD/CHF 0.95. This is because the value of a forward contract is of zero on its start date. On the maturity date the USD 750'000 sold have a value of 750'000 x 0.973 = CHF 729'750, so that the forward contract results in a loss of CHF 25'118.

The performance of the total investment is of 4.8981% because of an initial value of CHF 950'000 and a final value of CHF 996'525. This performance can also be calculated as the performance of the combination of the investment A and the forward contract, separated in a positive and negative part, ((950'000 x 7.5421) + (-699'387 x 4.3414) + (699'387 x 0.75)) / 950'000 = 4.8981%.

It is common to say that the performance of the forward is of -3.5914%, considering that it is an investment of CHF 699'387. The formula of the performance as a combination of investments gives us a correct value for the performance of the total investment.

However, it is important not to forget that this is only a convention of presentation for a forward's performance, since the investment itself is of zero and not of CHF 699'387. This presentation simply allows to know the amount of the positive and negative investments in both involved currencies.

In practice, the performance calculation of the forward contract is generally based on the notional amount of 704'632 instead of 699'387, which provides a performance of -3.5647% for the forward, based on a loss of CHF -25'118 and an invested capital of CHF 704'632. The only change is the basis of the calculation of the initial investment, since clearly there are no change in the realized gain or loss. The performance of the total investment can be calculated as previously demonstrated. The biggest advantage of this basis for the calculation is that the knowledge of the interest rates is unnecessary. The performance can be calculated based only on the transactions and the market values of the holdings, without having to know all interest rates for the involved maturity date.

We showed how to calculate the performance of a forward currency contract on its maturity date, but this method can also be applied at any time, as long as the value of the forward (unrealized gain or loss) is known or, equivalently, the forward rates on maturity date. In combining the calculation of the performance for more forwards it is possible to calculate and interpret easily the performance of a currency overlay.

This type of performance calculation for forward exchange transactions shows us how to define the benchmark for these forward exchange transactions. It is a composite benchmark with a positive (long) position in the purchased currency

and a negative (short) position in the currency sold, with the two positions having the same absolute weighting. The two items are, of course, translated into the base currency of the investment. Each of the two positions runs according to the interest rates of the corresponding currency. These interest rates should correspond to the rollover periodicity of the forward exchange transactions, for example, the 3-month interest rates if the forward exchange transactions are normally concluded for 3 months.
For our example, the following results are obtained:

	Currency	Index at 31.12.2012	Index at 31.12.2013	Perform. in %
Position USD (short)	USD	100	101.875	
	CHF	95	99.124	4.3414
Position CHF (long)	CHF	100	100.750	0.7500
Benchmark total	CHF			-3.5914

This benchmark performance is exactly the same as the calculated performance of the forward currency contract.

The benchmark of the forward currency contract can then be integrated into the composite benchmark of the overall investment, whereby its weighting is defined as a proportion of the value of the overall investment. For example, if the investment in USD represents 40% of the total investment and this investment needs to be hedged to 80%, the benchmark weighting of the forward currency contracts in USD is 32%. This example shows how the benchmark for partial hedging can be defined without any problems.

Even if the currency hedging is 100%, the benchmark of the investment in local currency must not be taken as a benchmark of the (currency) hedged investment, as this performance may not necessarily be achieved. In the above example, even if the benchmark performance of the investment is 5% in USD (the same as the performance of the investment itself), the performance with 100% currency hedging is 3.9507% and not 5%. This difference results from the different interest rates of the two currencies.

FX forwards in a currency overlay

We have seen how the performance of an FX forward is calculated and we now see how the performance of a group of FX forwards is calculated in a currency overlay. We have analyzed the forward sale of a foreign currency against the base currency, in this case the CHF. In a currency overlay, the exposure of a currency is dynamically adjusted, which can lead to the forward purchase of a foreign currency against the base currency or to a forward transaction with two foreign currencies. We will see that these forward exchange transactions must be treated differently from the forward exchange transactions seen so far.
In order to present the calculations as simply as possible, we use the simplified method for calculating forward exchange transactions, with an undiscounted nominal amount. The difference in the results is marginal and the discounted variant of the results can be calculated by interested persons without great difficulty.

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FX forward with a purchased foreign currency against the base currency

In this example, we have an initial investment of USD 1'000'000 on 31.12.2012 and a CHF/USD fx forward that reduces the exposure to the USD with a forward sale of USD 750’000. This exposure to the USD is then increased with a forward purchase of USD 300’000. Both forward exchange transactions take place here simultaneously, which would not be the case in practice. But this trick improves our understanding of the mechanism for increasing foreign currency exposure without us having to delve into complex calculations. The second forward exchange transaction consists of a positive (long) position in foreign currency (USD) and a negative (short) position in the base currency (CHF). The nominal amount used to calculate performance is the amount invested in base currency, which is negative in this case. The spot rate USD/CHF is 0.95 on 31.12.2012 and 0.973 on 31.12.2013. All results on 31.12.2013 are shown in this table:

Example

Investment	CCY	Value on 31.12.2012	Value on 31.12.2013	Perform. in %
Investment A	USD	1'000'000	1'050'000	5.0000
	CHF	950'000	1'021'650	7.5421
- FX FWD 1 USD (short)	USD	-750'000	-750'000	
	CHF	-704'632	-729'750	
- FX FWD 1 CHF (long)	CHF	704'632	704'632	
FX Forward 1 total	CHF	0	-25'118	-3.5647
- FX FWD 2 CHF (short)	CHF	-281'853	-281'853	
- FX FWD 2 USD (long)	USD	300'000	300'000	
	CHF	281'853	291'900	
FX Forward 2 total	CHF	0	10'047	-3.5647
Total Investment	CHF	950'000	1'006'579	5.9557

Both FX forwards show the same negative performance, as we have seen in the analysis of negative positions. The weighting of one FX forward is positive and the weighting of the other is negative, resulting in a negative performance contribution for the first and a negative performance contribution for the second.

The performance of the whole investment is 5.9557% = (1’006’579 – 950’000) / 950’000, which can also be calculated as follows: ((950’000 x 7.5421) - (704’632 x 3.5647) + (281’853 x 3.5647)) / 950’000 = 5.9557%. The performance of the FX forward 1 is weighted by the positive nominal amount of the forward sale of the foreign currency and that of the FX forward 2 by the negative nominal amount of the forward purchase of the foreign currency.

FX forward with two foreign currencies

We are now examining an FX forward with two foreign currencies, here EUR and USD, whereby the base currency remains CHF. The decisive point is the consideration of the EUR/USD FX forward (purchase of EUR against sale of USD) as a combination of a EUR/CHF FX forward (purchase of EUR against sale of CHF) and a CHF/USD FX forward (purchase of CHF against sale of USD),

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whereby the CHF amount is the same for both fictitious FX forwards. If it were not the case, there would be an arbitrage opportunity. The nominal amount added for the performance calculation is then negative for the EUR/CHF FX forward and positive for the CHF/USD FX forward, as we have seen in the chapter above. If we consider both FX forwards together, the added nominal amount is zero. Let us now consider a concrete example, which is, unfortunately, somewhat complex by nature.

Example

The initial investment as at 31.12.2012 consists of a USD 1’000’000 investment, a EUR 600’000 investment, a CHF/USD FX forward for USD 700’000, a CHF/EUR FX forward for EUR 300’000 and an FX forward for the purchase of EUR 100’000 against the sale of USD 118’762.

The spot rates on 31.12.2012 are 0.95 for the USD/CHF, 1.12 for the EUR/CHF and therefore 1.1789 for the EUR/USD. The forward rates result from the following interest rates: 0.75% for the CHF, 2.75% for the USD and 2.00% for the EUR, which results in the following 1-year forward exchange rates: 0.9315 for the USD/CHF, 1.1063 for the EUR/CHF and 1.1876 for the EUR/USD.
The spot rates on 31.12.2013 are 0.973 for the USD/CHF and 1.08 for the EUR/CHF.

Investment	Ccy	Value on 31.12.2012	Value on 31.12.2013	Perform. in %
Investment USD	USD	1'000'000	1'050'000	5.0000
	CHF	950'000	1'021'650	7.5421
Investment EUR	EUR	600'000	580'000	-3.3333
	CHF	672'000	626'400	-6.7857
- FX FWD 1 USD (short)	USD	-700'000	-700'000	
	CHF	-652'056	-681'100	
- FX FWD 1 CHF (long)	CHF	652'056	652'056	
FX FWD 1 total	CHF	0	-29'044	-4.4542
- FX FWD 2 EUR (short)	EUR	-300'000	-300'000	
	CHF	-356'285	-324'000	
- FX FWD 2 CHF (long)	CHF	356'285	356'285	
FX FWD 2 total	CHF	0	32'285	9.0615
- FX FWD 3 USD (short)	USD	-118'762	-118'762	
	CHF	-110'627	-115'555	4.4542
- FX FWD 3 EUR (long)	EUR	100'000	100'000	
	CHF	110'627	108'000	-2.3750
FX FWD 3 total	CHF	0	-7'555	-6.8293
Whole Investment	CHF	1'622'000	1'643'736	1.3401

Note that the CHF value of USD 118,762 and EUR 100,000 over one year for the FX forward 3 are equal and amount to CHF 110,627.

The performance of the whole investment is 1.3401% = (1'643'736 - 1'622'000) / 1'622'000 and can also be calculated using the following formula ((950'000 x 7.5421) - (672'000 x 6.7857) - (652'056 x 4.4542) + (356'285 x 9.0615) - (110'627 x 4.4542) - (110'627 x 2.3750)) / (950'000 + 672'000) = 1.3401%. In this formula, the FX forward 3 was divided into two fictitious FX forwards with CHF as the counter currency of the respective foreign currency, as we have seen in the above chapter.

A detailed analysis of the investment requires a separate analysis of the investment without currency hedging and the currency hedging itself, i.e. the "currency overlay". The calculation of the system without currency hedging is simple and results in a value of 1.6060%. The calculation of the performance of the currency overlay is given by the following formula: (-(652'056 x 4.4542) + (356'285 x 9.0615) - (110'627 x 4.4542) - (110'627 x 2.3750)) / (652'056 + 356'285) = -0.4279%.

We can aggregate these two parts in order to calculate the performance of the whole investment differently: (((950'000 + 672'000) x 1.6060) + ((652'056 + 356'285) x -0.4279)) / (950'000 + 672'000) = 1.3401%. Note that the formulas are not intuitive if the FX forward 3 is not divided into its two components. With this division, on the other hand, they become simple and self-explanatory.

With a currency overlay it is indispensable to calculate the exposures for the different currencies. This calculation can be easily derived from the investment data. The following table shows the currency exposures of our example on 31.12.2012:

Investment	CHF	USD in CHF	EUR in CHF	Total in CHF
Investment USD	0	950'000	0	950'000
Investment EUR	0	0	672'000	672'000
FX FWD 1 CHF/USD	652'056	-652'056	0	0
FX FWD 2 CHF/EUR	356'285	0	-356'285	0
FX FWD 3 EUR/USD	0	-110'627	110'627	0
Total (CHF)	1'008'341	187'317	426'342	1'622'000
Total (%)	62.17	11.55	26.28	

Futures

A future contract is a commitment to deliver a fixed quantity of an underlying asset on a fixed date (maturity date). Futures are negotiated on an organized futures market. On the maturity date, the future results in either a gain or a loss, depending on whether the price of the underlying asset on the maturity date is lower or higher than the price of the future at the date of the commitment. The calculation of the performance with the simple performance formula is not possible since the initial investment is zero. It is only a commitment resulting of a gain or a loss on the maturity date.

An example is given by the futures on the SMI (Swiss Market Index) of the most important capitalizations of the Swiss stock exchange. Those futures are standardized with a maturity date on the third Friday of March, June, September and December. The contract has a factor of 10, which means that one futures contract is valid for the purchase or sale of 10 SMI indexes. With these futures it is possible to bet on the raise (purchase of futures) or the drop (sale of futures) of the SMI index. Holdings in Swiss stocks, for example, can be covered against a drop by selling SMI futures.

The booking of futures by the banks is not unified, but often uses a margin account which we will ignore for the calculation of the futures' performance. We calculate the performance of the futures by considering an equivalent investment to the purchase of futures, which means producing the same gains and losses as the futures themselves. For simplicity reasons we suppose that no dividend income occurs during the period of calculation. This similar investment is made of a loan and a purchase of an underlying asset corresponding exactly to the borrowed amount. The net result of this engagement is of zero, just as the futures themselves. It only remains to determine the borrowed amount which was invested in an underlying asset. This amount, increased with its interests until the maturity date, has to be equal to the amount invested in the futures, which means corresponding to the price of the futures on the engagement day. It is easy to see that a gain or a loss of the investment is the same as a gain or a loss on the futures themselves. We will now calculate the performance of the futures as a combination of the positive holding ("long") in the underlying and of the negative holding ("short") in cash as defined above. Since the invested amount is the same for both holdings, once positive and once negative, we can define the performance of the futures as the performance of the positive holding minus the performance of the negative holding, in compliance with the formula of performance calculation for a combination of investments. Note that it is imperative to calculate the performance of the negative holding as explained above. The invested capital is considered as being the absolute value of the initial capital for both holdings.

The sale of futures can be analyzed similarly, the similar investment consisting of a short sale of the underlying and the cash investment resulting from this sale.

Example

On January 1st we invest CHF 1'000'000 in Swiss stocks which we want to protect against a drop with the sale of futures for CHF 800'000 with a maturity date on December 31st. The interest rate for a year is 2%. We will calculate the performance of the total investment and its components for the current year.

On December 31st our Swiss stocks have a value of CHF 920'000 and the nominal value of the futures is of CHF 725'490. All the results on that date are represented in the following table:

Investment	Value on January 1st	Value on December 31st	Perform. in %
Swiss stocks	1'000'000	920'000	-8.0000
- Futures part in Swiss stocks (short)	-784'314	-725'490	-7.5000

- Futures part in cash (long)	784'314	800'000	2.0000
Future total	0	74'510	9.5000
Total investment	1'000'000	994'510	-0.5490

The invested amount of CHF 784'314 gives a value on December 31st of CHF 800'000, including the interests of 2%. The Swiss stocks resulted in a performance of -8% for the year. The value of the investment (negative) in the underlying is of CHF -725'490. Note that this value is also the nominal value of the futures since we have reached the maturity date. We can therefore calculate the performance of the equivalent investment as the difference between the cash part of the investment (2%) and the stocks part of the investment (-7.5%), which means a performance of 9.5%, also the performance of the futures. The performance of the total investment can then be calculated directly from its beginning and end values which results in -0.549%. This performance can also be calculated as the performance of the combination of the investment in Swiss stocks and futures, separated in a positive part and a negative part: ((1'000'000 x (-8)) + (-784'314 x (-7.5)) + (784'314 x 2) / 1'000'000 = -0.549%.

It is common to say that the performance of the future is of 0.549%, considering that it is an investment of CHF 784'314. The formula of the performance as a combination of investments gives us a correct value of the performance of the total investment. However, it is important not to forget that this is only a convention of presentation for a future’s performance, since the investment itself is of zero and not of CHF 784'314. This presentation simply allows to know the amount of the positive and negative investments in both parts of the equivalent investment.

In practice the performance calculation of the future contract is generally based on the notional amount of 800'000 instead of 784'314, which provides a performance of 9.3138% for the future, based on a gain of CHF 74'510 and an invested capital of CHF 800’000. The only change is the basis of the calculation of the initial investment, there are of course no change in the realized gain or loss. The performance of the total investment can be calculated as previously demonstrated. The biggest advantage of this basis for the calculation is that the knowledge of the interest rates is unnecessary. The performance can be calculated based only on the transactions and the market values of the holdings, without having to know all interest rates for the involved maturity date.

We demonstrated how to calculate the performance of a future contract on its maturity date, but this method can also be applied at any time, as long as the value of the future is known. This way it is possible to calculate the performance of the combination of several futures.

Cash Accounts

At first it might seem surprising to see a chapter of this document dedicated to cash accounts, but cash accounts have properties which make the calculation of their performance difficult, even impossible under certain circumstances. The different difficulties could be listed as follow:

- Important capital flows which could be very high compared to the balance of the account.
- Periodically debited or credited interests, which can hardly be calculated daily, especially with changes of rates, reversals, etc. …
- Periodical fees, which can also hardly be calculated daily.
- Moving from a positive balance to a negative balance and vice versa.

Example 1

A cash account has a balance of CHF 10 at the beginning of the period, fees of CHF 20 during the period, which results in a balance of CHF -10 at the end of the period. What is the performance of this cash account during the period? No answer is satisfactory nor instinctive.

Example 2

This second example shows the effects of a very high capital flow compared to the balance of the account, leading to important distortions. At the beginning of the period we have an account in USD with a balance of USD 100 and a value of CHF 94.50. On the same day an amount of USD 100’000 with an exchange rate of 0.945 resulting from the sale of assets is credited to the account, then an amount of USD 99’500 with an exchange rate of 0.943 resulting from the purchase of assets is debited from that same account. Finally, the balance of USD 600 with an exchange rate of 0.944 represents a value of CHF 566.40. These informations are summarized in the following table:

	USD	USD/CHF	CHF
Balance transfer	100.00	0.9450	94.50
Sale of assets	100'000.00	0.9450	94'500.00
Purchase of assets	-99'500.00	0.9430	-93'828.50
Balance	600.00	0.9440	566.40

The sum of the values in CHF shows a loss of 199.40 with an initial value of 94.50. The calculation of the performance, independently of the used method, gives a result which is difficult to interpret.

These 2 examples clearly demonstrate that the calculation of the performance for cash accounts can lead to results which have no intuitive interpretation and should be avoided in order to prevent the presentation of results which have no practical interpretation.

Performance of the total investment, of sub-portfolios and individual securities

We just covered how to calculate the performance of different investment types, in more or less complex situations. We will now analyze for which investments the performance can be measured and which are the required data for this calculation. If we disregard derivative products, some of which have been analyzed specifically in this document, the required data for the performance calculation for a given period are the market value at the beginning and at the end of the period (including accrued interests) and the capital flows during the period. It is then possible to calculate the performance not only for the total investment, but also for every asset group of this investment. The calculation of the market value at the beginning and the end of the period can be done without any difficulty for every asset group. It is however important to be more cautious concerning the capital flows. Let's take as an example the purchase of an asset. At the level of the whole investment, no capital flow has occurred, since the amount which went out of the cash account is invested in the purchased asset. The situation can look totally different for a performance group. Let's suppose that the purchased asset belongs to the performance group, but not the cash account. The purchase will then generate an incoming capital flow corresponding to the amount of the purchase. The sale of this asset obviously results in an outgoing capital flow from the performance group. Similarly, a dividend income is an outgoing capital flow from the performance group if the asset belongs to it, but not the used cash account.

It is important to realize that a performance group could have important capital flows, even if these flows are negligible at the level of the whole investment. It is important to consider these flows in order to correctly interpret the results, as shown in the previous example.

The performance of a group is always calculated anew, independently of the performance of the other groups. This insures that a problem in the performance calculation of a group will have no impact on the performance calculation of another one, particularly on the calculation of the performance of the whole investment.

Which performance groups have a special relevance? The answer to this question naturally depends on the investment itself, but we can give certain indications.

- The asset classes themselves can be split to provide more transparency in the performance analysis. A foreign equities asset class, for example, can be divided into equity asset classes per country. The most important countries can then be analyzed as separate performance groups. Comparing the performance of these groups with a corresponding benchmark then increases the transparency of the analysis. In addition, the benchmarks for equities in individual countries are often more readily available and cheaper than the composite benchmarks (multi-country) of specialized institutions.

- The analysis of the positions per currency can be particularly important, especially for fixed-interest investments. Again, the availability of benchmarks for each currency is higher than the multi-currency benchmarks of specialized institutions. It is even possible to calculate the performance in the respective currency (the market values and capital flows in this currency are readily available) and compare it with a benchmark in this currency so that exchange rate fluctuations can be neutralized.

- The performance calculation is also possible at the level of individual securities. However, it is important to realize that small inaccuracies have bigger consequences on a security level than on the level of a bigger performance group, such as the whole investment. Let's take the example of a dividend income. It is recorded on the date of its payment instead of the "ex" date of the dividend, which leads to a distorted performance between these 2 dates at the security level, but would generally be marginally distorted at a larger group level. Let’s also keep in mind that the TWR performance at the security level neutralizes practically all transactions on this asset, except price variations within a day for purchases and sales. Therefore, the TWR performance of an asset gives basically no information about the success of the portfolio manager concerning the security. The example at the end of this chapter shows it perfectly.

- The monitoring of the individual securities is particularly interesting for the investment funds. Their development can be continuously compared with the corresponding benchmark. If the investment consists of a mixture of direct and indirect investments, a performance group can be defined with all direct investments and compared with the performance of the investment funds in the same asset class.

- The performance groups considered up to now are static groups: the assignment of an asset to a performance group does not change over time. However, it may be advantageous or even necessary to define dynamic performance groups for which the assignment of an asset can change over time. A performance group of investment grade bonds, for example, will gain or lose an asset depending on the development of its rating. Another example can be given with Swiss stocks belonging to the SMI (Swiss Market Index). The composition of this index is periodically adjusted and changes the performance group accordingly. The inclusion of a security in the group must be simulated with an "artificial" cash flow that corresponds to the market value of the security at the time of inclusion. Similarly, the exclusion of a security from the group must be simulated with an "artificial" cash flow that corresponds to the market value of the security at the time of exclusion.

The examples of performance groups given here are of course not exhaustive. You can even define a performance group by selecting the titles of this group.

Example of the performance of a single security

Let’s consider an asset with the following transactions (trx) and daily prices:

Date	Trx	Qty.	Price trx	Amount trx	Price end day	Value end day
01.03.2013	Purchase	10	55	550	55.4	554
15.04.2013	Purchase	10	42	420	41.8	418
05.05.2013	Purchase	10	39	390	39.2	392
30.06.2013	Sale	30	51	1’530	-	-

This asset’s TWR performance for the period from 01.03.2013 until 30.06.2013 is negative. This is directly visible since its price went from 55.4 to 51 during the period. The exact calculation gives a performance TWR of -7.04%. If, however, we look at the results of the absolute investment, we get a totally different picture. The net result of this investment is 170 = 1'530 -550 - 420 – 390, since the manager bought a part of these assets for a price which was lower than the price of June 30th 2013. This result is reflected in the MWR performance of this investment which is of over 20%.

It is therefore important to be wary of a fast interpretation of the TWR performance of a single security.

Performance Contribution

When we analyze the performance of an entire investment, we want to know where this performance comes from. Hence the question: how have assets or groups of assets contributed to this performance? Usually there are clearly defined asset classes, such as "Bonds in CHF", "Foreign Stocks", "Swiss Real Estate", etc. and we assume that the performance of these asset classes is known. However, an asset class with a very high or very low performance has only a small impact on the performance of the overall investment if the weighting of this asset class itself is low.

Consequently, we take this weighting into account to determine the Performance Contribution of an asset or a group of assets.

Simple example

Investment with 3 asset classes and no transaction.

	Value on 31.12.2013	Value on 31.12.2014	Perform. 2014 in %	Contribution 2014 in %
Asset class A	200	208	4.0	0.8
Asset class B	300	294	-2.0	-0.6
Asset class C	500	512	2.4	1.2
Whole investment	1'000	1'014	1.4	1.4

Asset class A has a weighting of 20% (200 / 1’000) at the beginning of 2013 and a performance of 4% for 2013, resulting in a performance contribution for 2013 of 0.8% (= 4% x 20%). The calculation for the other asset classes is similar.

We can therefore say that asset class A contributed 0.8% to overall performance in 2013, asset class B -0.6% and asset class C 1.2%, giving a total of 1.4%, which corresponds to the performance of the overall investment.

The definition and calculation of performance contribution becomes much more complex when there are capital flows between asset classes.

Example with a capital flow between asset classes

Let us take the previous example and assume that a sum of 50 will be transferred from asset class C to asset class A at the end of 2013.

	Value on 31.12.2014	Value on 31.12.2015	Perform. 2015 in %	Contribution 2015 in %
Asset class A	258	269	4.2636	1.0848
Asset class B	294	305	3.7415	1.0848

Asset class C	462	456	-1.2987	-0.5917
Whole investment	1'014	1'030	1.5779	1.5779

The calculation of the Performance Contributions for 2015 is similar to that for 2014, so let's look at the two years 2014 and 2015 together. The table below simply shows the performance contributions of the two years:

	Perform. 2014 in %	Perform. 2015 in %	Perform. 2014-2015 in %	Sum of Contribution 2014-2015 in %
Asset class A	4.0	4.2636	8.4341	1.8848
Asset class B	-2.0	3.7415	1.6667	0.4848
Asset class C	2.4	-1.2987	1.0701	0.6083
Whole investment	1.4	1.5779	3.0000	2.9779

Of course, the sum of the contributions does not equal the overall performance for the two years. We know that the performance of the years combined does not equal the sum of the performance of the two years.

Idea for a correction

We adjust the performance contributions of the asset classes for 2014 with the overall performance achieved in 2015, as shown in the following table:

	Contribution 2014 in %	Adjusted Contribution 2014 in %	Contribution 2015 in %	Contribution 2014-2015 in %
Asset class A	0.8	0.8126	1.0848	1.8974
Asset class B	-0.6	-0.6095	1.0848	0.4753
Asset class C	1.2	1.2189	-0.5917	0.6272
Whole investment	1.4		1.5779	3.0000

It is a possible method for calculating the performance contributions of asset classes over several periods, but it is not the only one. In order to calculate the performance contributions of the asset classes, the calculation period must be divided into small time intervals without transactions. If necessary, the calculation period must be divided into days.

Performance contribution of negative (short) positions

In an earlier chapter, we saw that the performance of a negative position is counterintuitive at first glance: it is positive for a loss and negative for a profit. With Performance Contribution, we understand this paradox a little better, since performance contribution changes the sign of performance by multiplying it by the weight. We then have a positive Performance Contribution on a profit and a negative Performance Contribution on a loss, which corresponds to our intuition.

Performance Attribution

As a rule, the allocation into asset classes does not only consist of a list of these asset classes to be analyzed, but also of an investment strategy. This means that each asset class is assigned a (strategic) weight and a benchmark.

The role of the investment strategy is essential, as it has a decisive influence on the performance of the investment on the one hand and limits the risk of the investment by limiting the investment in certain asset classes and by taking into account the correlations between these asset classes.
Example of a strategic weighting of asset classes:

Asset classes	Weight in %
Liquidity	2
Bonds CHF	35
Bonds foreign currency	10
Stocks CH	15
Stocks foreign	20
Real Estate CH	10
Alternative investments	8
Total	100

We can use the investment strategy to analyze the overall performance over a given period, or more precisely, to break it down. We first define the strategic performance that would be achieved if we invested exactly in accordance with the strategic weighting and with a precise mapping of the corresponding benchmarks. In our example, exactly 15% of our investment would be invested in Swiss equities, exactly according to the weighting of the SPI, assuming that the SPI is our benchmark for Swiss equities. The goal of performance attribution is to analyze the difference between realized performance and strategic performance.

In fact, the actual performance of our investment will differ from the strategic performance and we want to explain this difference. There are two main reasons for this difference: firstly, the effective weighting of our investment will differ from the strategic weighting and secondly, the selection of securities in each asset class will differ from the corresponding benchmark. The first component of the difference is called "allocation" and the second "selection". A third component remains that cannot be uniquely assigned to one of the mentioned components and is called "interaction". We now see the mathematical formula of this decomposition, which bears the name of its authors: Brinson, Hood & Beebower.

First let us define the following variables:

w_i^P	Weight of asset class i at the beginning of the period
w_i^B	Strategic weight of asset class i
r_i^P	Performance of asset class i
r_i^B	Performance of the benchmark of asset class i

The performance R of the whole investment is given by:

$$R = \sum_i w_i^P r_i^P = \text{¿} \sum_i w_i^B r_i^B + \sum_i \left(w_i^P - w_i^B\right) r_i^B + \text{¿} \sum_i w_i^B \left(r_i^P - r_i^B\right) + \sum_i \left(w_i^P - w_i^B\right)\left(r_i^P - r_i^B\right) \text{¿¿}$$

We recognize the following components in the above formula:

$\sum_i w_i^B r_i^B$	The strategic performance
$\sum_i \left(w_i^P - w_i^B\right) r_i^B$	The «allocation» component, resulting from the deviation from the effective weight w_i^P of the asset class i from the strategic weight w_i^B of this asset class.
$\sum_i w_i^B \left(r_i^P - r_i^B\right)$	The «selection» component, resulting from the deviation of the effective performance r_i^P of the asset class i from the performance of the corresponding benchmark r_i^B.
$\sum_i \left(w_i^P - w_i^B\right)\left(r_i^P - r_i^B\right)$	The «interaction» component which cannot be assigned to one of the above components.

The above formula shows that the results of the performance attribution can be presented not only at the level of the overall investment, but also for each asset class. The performance components (strategy, allocation, selection and interaction) can therefore be assigned to the individual asset classes. The following table gives an example of the results of a performance attribution:

Asset class	Effect. wght w_i^P	Strat wght w_i^B	Effect Perf. r_i^P	Perf. Bench r_i^B	Contr Perf. $w_i^P r_i^P$	Contr. Strat. $w_i^B r_i^B$	Allocation	Selec-tion	Inter-action
Liquidity	14.01	5.00	0.21	-0.05	0.05	0.00	0.00	0.01	0.04
Bonds CHF	28.96	32.00	3.83	4.21	1.09	1.35	-0.15	-0.12	0.01
Bonds foreign ccy	4.43	3.00	3.98	-1.11	0.16	-0.03	0.01	0.14	0.04
Stocks CH	12.72	13.00	18.10	17.72	2.26	2.30	-0.08	0.05	0.00
Stocks	11.20	15.00	11.60	13.37	1.29	2.01	-0.54	-0.26	0.08

foreign									
Real estate CH	11.40	12.00	6.72	4.00	0.77	0.48	-0.02	0.33	-0.02
Real estate foreign	3.03	5.00	5.99	4.43	0.17	0.22	-0.09	0.08	-0.04
Alternative investment	10.22	15.00	0.02	1.18	-0.21	0.18	-0.27	-0.16	0.05
Capital protection	4.03	0.00	1.45	0.00	0.07	0.00	0.00	0.00	0.07
Currency Overlay*)	0.00	0.00	1.08	1.07	0.10	0.11	0.02	0.00	-0.03
Total	100	100	5.76	6.62	5.76	6.62	-1.13	0.08	0.19

*) The treatment of a currency overlay is analyzed in a later chapter, in particular with regard to its weighting.

Brinson & Fachler

With the Brinson, Hood & Beebower model presented above, the allocation component of an overweighed asset class always becomes positive if the performance of the assigned benchmark is positive, even if this performance is lower than the performance of the overall benchmark. Similarly, the allocation component of an overweighed asset class always becomes negative if the performance of the assigned benchmark is negative, even if this performance is greater than the performance of the overall benchmark. To correct this inaccuracy, Brinson & Fachler proposed a replacement for the allocation component:

Brinson, Hood & Beebower	Brinson & Fachler
$\sum_i \left(w_i^P - w_i^B\right) r_i^B$	$\sum_i \left(w_i^P - w_i^B\right)\left(r_i^B - r^B\right)$

where r^B is the performance of the overall benchmark. Since the sum of the effective weightings w_i^P and the benchmark weightings w_i^B are both 1, we have left the sum of the allocation components unchanged, although we have changed each one individually.

The performance attribution according to Brinson & Fachler is often preferred to the Brinson, Hood & Beebower model.

Difficulties of the performance attribution

The apparent simplicity of the performance attribution formula is somewhat deceptive; we now look at the hidden difficulties. The main problem is the weighting of asset classes. In the upper formula, the weighting is used at the beginning of the calculation period. The following question then arises: "Can the performance attribution be calculated and displayed over a period of any length?", although transactions can change the weighting of asset classes at any time. As we have seen for performance contribution, the answer is negative. The performance attribution must be calculated for shorter time intervals and aggregated over the entire calculation period. This additional step brings with it additional difficulties, not only for the calculation of the performance attribution, but also for its interpretation.

In fact, an asset class with exactly the strategic weighting at the beginning of a time interval gets a different weighting at the end of that time interval, even if no transactions have taken place during that time interval. This is due to market developments, which are different for each asset class. These deviations from the strategic weighting are then called over- or underweighting of the asset classes for the next time interval and therefore generate an allocation component that would be omitted if the performance attribution were calculated for the entire calculation period. A "neutral" performance attribution then assumes a reallocation for strategic weighting at the beginning of each time interval. A selection component of a time interval also causes an allocation component in the next time interval, because it causes a different weighting at the beginning of this next time interval. The distinction between allocation and selection components is then not as clear as the performance attribution formula suggests.

Aggregating the results of the short time intervals over the entire calculation period poses a further challenge. As we have seen for performance contribution, the results of the time intervals cannot simply be added together. A similar method can be used as for performance contribution (the results of a time interval are multiplied by the global performance of the investment from the end of the time interval to the end of the entire calculation period). However, there are numerous other methods and none can be described as "standard".

Strategic bandwidth

We have mentioned two elements for the definition of the investment strategy: the weighting of the asset classes and the associated benchmarks. Actually, there is an additional element, the bandwidth. This defines a minimum and a maximum weighting for each asset class. These ranges give the asset manager scope and allow him to over- or underweight certain asset classes. Thanks to the bandwidths, the asset manager does not have to react constantly to market changes and carry out a reallocation for strategic weighting. It is therefore almost impossible to know whether an overweighting or underweighting of asset classes results from an active decision by the asset manager or simply from market developments. The consequences of market developments for performance attribution are regarded as active decisions by the asset manager, which does not necessarily correspond to the investor's intuition.

Consistency and aggregation of performance attribution results

In a period without transaction, the performance contribution of asset class i is actually given by $w_i^P r_i^P$ and we can break this contribution down into its 4 additive components: strategy, allocation, selection and interaction. In addition, the sum of the performance contributions of the asset classes is equal to the performance of the entire investment. We therefore have perfectly consistent results in the performance attribution table as in the example above.

However, if there are transactions in the calculation period, the performance contribution of asset class i is not equal to $w_i^P r_i^P$ and the sum of the performance contributions $w_i^P r_i^P$ is not equal to the performance of the entire investment. In this case we can present a table whose values are not perfectly consistent or insert residual values or "adjust" the calculated values. Note that the inconsistencies are usually small when capital flows between asset classes are small compared to the market value of these asset classes.

If the calculation period is divided into small time intervals, the results of the individual time intervals must be aggregated to present the performance attribution for the entire calculation period. There are several such aggregation methods that maintain the consistency of the results of the time intervals without there being a "standard". For example, mention the Frongello method, which multiplies the results of a time interval by the global performance of the investment from the end of the time interval to the end of the entire calculation period. This is essentially the method we used to calculate the multi-period performance contribution.

It may be advantageous to supplement the performance attribution results with the MWR performance of the individual asset classes. A large difference between TWR and MWR performance of an asset class can indicate that large capital flows have had a significant impact on performance and can thus avoid incorrect interpretation of performance attribution results.

In summary, we are far from the apparent simplicity of performance attribution and the interpretation of the results is a delicate task.

Performance attribution with a currency overlay

The performance attribution presented here can also be calculated and presented with a "currency overlay". But the definition of the weighting of the "currency overlay" asset class must be adjusted. The weighting of the currency overlay is not only based on its market value, but also on its market value and its hedging amount, as we have seen in the chapter on the performance of currency overlay. Of course, the calculation of the performance of the currency overlay must take the hedging amount into account, as mentioned in the chapter above. The sum of the weightings of the asset classes with the "currency overlay" is then not 100%! The reason for this is that the weighting from the hedging amount of the FX forwards can be seen as a combination of a positive (long) and negative (short) investment with the same value. The weighting from the hedging amount then disappears and only the weighting from the market value remains, which results in a total of 100%.

Example of the calculation of the weighting

On a certain date we have an investment of USD 1’000’000 at a USD/CHF rate of 1.02 = CHF 1’020’000. On the same date, we also have an FX forward for the sale of USD 500’000 against CHF 498’000, with a market value of CHF 12’000. The total market value is then 1’020’000 + 12’000 = CHF 1’032’000.

The weighting of the USD investment is then 1'020'000 / 1'032'000 = 98.84%. The weighting of the FX forward is (12’000 + 498’000) / 1’032’000 = 49.42% = 1.16% + 48.26%. This weighting includes a market value component of 1.16%

and a hedging component of 48.26%, corresponding to the decomposition of the forward exchange transaction as a negative position in USD of -48.26% and a positive position in CHF of 48.26%. The sum of the weights is then 100%.

Transaction-based performance attribution

The performance attribution method presented above is based on the holdings at the beginning of the period (holdings based performance attribution). As we have seen, this method can give difficult to interpret results, as the sum of the performance contributions of the different asset classes does not correspond to the performance of the overall investment. This inequality can be alleviated but not completely removed with transaction-based performance attribution. This method uses the same formulas as the holdings-based performance attribution, but the weighting used depends not only on the positions at the beginning of the period, but also on the transactions during the period, or rather, on the cash flows that these transactions cause. This improves the accuracy of the method because, as we have already seen, these cash flows affect the performance of the period. Since the capital of these flows is not available or is withdrawn at the beginning of the period, this capital is weighted by a factor that reflects the portion of the period in which the capital is available. The sum of the capital at the beginning of the period and the weighted cash flows during the period is the average invested capital. Note that if there are no transactions during the period, the average invested capital is equal to the capital at the beginning of the period. In this case, the holdings-based and transaction-based performance attribution give the same results.

Example

We calculate the performance attribution for the month of April. For a particular asset class, we have an investment at the beginning of the period of CHF 10’000 and a capital inflow of CHF 3’000 on April 20. We weight this inflow by the number of days up to the end of the period (10) relative to the total number of days of the period (30) and get 3'000 x 10 / 30 = 1'000. the average invested capital is then 10'000 + 1'000 = CHF 11'000.

Modified Dietz performance and average invested capital

Let us again write the formula for calculating the MWR performance in a period $(0,T)$ with a capital at the beginning of the period of V_0, a capital at the end of the period of V_E and n capital flows F_i at the points in time t_i. The internal rate of return r is the solution of the equation

$$V_0(1+r)^T+\sum_{i=1}^{n} F_i(1+r)^{T-t_i}=V_E$$

We approximate this function by its Taylor time series of the first degree at the value $r=0$ and get

$$V_0+\sum_{i=1}^{n} F_i+r\left(T V_0+\sum_{i=1}^{n}(T-t_i)F_i\right)=V_E$$

The solution of this equation is given by the following formula

$$Tr = \frac{V_E - V_0 - \sum_{i=1}^{n} F_i}{V_0 + \sum_{i=1}^{n} \frac{T - t_i}{T} F_i}$$

where Tr is the performance on the period of length T. This performance is the so-called "modified Dietz" performance and usually gives a very good approximation of the IRR (internal rate of return).

The numerator of this formula is the difference between the capital at the end of the period, the capital at the beginning of the period and the cash flows during the period. This is the net result (absolute performance) of the investment in CHF. The denominator of the formula is nothing other than the average invested capital, as shown above. The division can then also be seen intuitively as the performance of the investment.

Example

We now present an example of the calculation of performance attribution using the two methods. The first is the holdings-based performance attribution and the second is the transaction-based performance attribution.

Asset class	Effective perform.	Holdings-based		Transaction-based	
		Weight	Contribution	Weight	Contribution
Liquidities	-0.2066	16.17%	-0.0334	16.00%	-0.0331
Bonds CHF	0.7387	26.09%	0.1927	27.04%	0.1997
Bonds foreign Ccy	-0.4939	4.88%	-0.0241	4.90%	-0.0242
Stocks CH	1.3857	13.08%	0.1812	13.07%	0.1812
Stocks foreign	3.3992	11.72%	0.3984	11.63%	0.3952
Real estate CH	0.2587	11.31%	0.0293	11.22%	0.0290
Real estate foreign	-0.8426	3.20%	-0.0270	3.19%	-0.0269
Alternative investments	-1.3668	8.25%	-0.1127	8.24%	-0.1127
Capital protection	0.0520	5.67%	0.0029	4.99%	0.0026
Currency overlay	1.5023	12.94%	0.1944	13.00%	0.1952
Whole investment	0.8061	100.00%	0.8018	100.00%	0.8061

We see in the table above that the sum of the performance contributions of the asset classes corresponds to the performance of the entire investment for the transaction-based performance attribution, which is not the case for the holdings-based performance attribution.

Performance Differences

Now that we have seen in details the required data in order to calculate the performance, we can now understand the reasons why the performance of a group for a given period can be different, depending on the source.

1. The asset list at the beginning or at the end of the period is not identical for both sources. The cause could be different transaction dates. For example, one system could provide an asset list based on the execution date, while another system could base its asset list on the value date. Another cause could be the different categorization of the assets, so that one security belongs to a performance group for a system, but to another one in another system. It could also be that an asset category such as for example forward currency contracts, are handled by a system but not in another.

2. Asset prices and/or exchange rates for the inventory at the beginning and at the end of the period are different in both systems.

3. Accrued interests for the inventory at the beginning and at the end of the period are different in both systems.

4. For the same reason as with the differences in the inventory, it is also possible that some transactions are considered on different dates for both systems. This could have an important impact for corporate actions.

5. The calculation method could differ from a system to another. For example, the TWR performance can be calculated on a daily basis by one system and on a monthly basis by another. The annualization of the results could also lead to differences.

6. How fees are handled can also differ from one system to another. The performance can be calculated before or after the fees. In the first case the fees will not have an impact on the performance, in the second case it will reduce it. We distinguish the transaction fees (brokerage, taxes, etc. …) and the other fees (management fees, flat fees, etc. …). The transaction fees can be included for all the performance groups. The other fees can normally only be included for the performance of the whole investment.

7. The management of taxes on interests and dividends can also differ from one system to another. The payment of interests and dividend income is often after deduction of a tax, which can be partly recoverable and partly not recoverable. The performance is generally calculated after the deduction of the non-recoverable taxes. Concerning the recoverable taxes, the performance can be calculated before its deduction, as if the taxes were recovered immediately or after its deduction. In the second case the transaction of tax recovery must be included in the performance calculation.